CSE 483: Mobile Robotics

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Linearization of the Probabilistic Observation Model

In the previous lecture, we linearized the robot's (probabilistic) motion model using Taylor Expansion. We observed that the linearization resulted in an *estimate* of the robot's next state. We now refine this estimate by incorporating the measurement, after the control has been applied.

1 Linearizing the Measurement

A mobile robot typically uses sensors to measure its state at a given time instant. Let us denote this measurement by $\mathbf{z_t}$. Since it may not be feasible to measure all the variables that represent a robot's state, the dimensionality of $\mathbf{z_t}$ need not be equal to that of $\mathbf{X_t}$. Here, we assume that $\mathbf{z_t}$ is made up of two components - r_t, the distance to a known landmark at time t, and ψ_t, the angle made by the robot (the mean position of the robot) with the known landmark. We also refer to ψ_t as the *bearing* and to r_t as the *range*.

$$\mathbf{z_t} = \begin{bmatrix} r_t \\ \psi_t \end{bmatrix} \tag{1}$$

In the above discussion, $\mathbf{z_t}$ is the actual measurement reported by, say, a laser rangefinder. However, based on $\mathbf{X_{t+1}}$, the estimate of the robot's state at time $t+1$ that we obtained using a linearization of the motion model, we have an estimate of the measurement $\hat{z}_{t+1}$.

$$\mathbf{\hat{z}_{t+1}} = \begin{bmatrix} \hat{r}_{t+1} \\ \hat{\psi}_{t+1} \end{bmatrix} \tag{2}$$

Also, $\mathbf{z_{t+1}}$ is the actual measurement at time $t+1$ as measured by the laser rangefinder.

$$\mathbf{z_{t+1}} = \begin{bmatrix} r_{t+1} \\ \psi_{t+1} \end{bmatrix} \tag{3}$$

We further assume that the coordinates of the known landmark in the global frame are (m_x, m_y). Then, $\mathbf{\hat{z}_{t+1}}$ is given by the following equation.

$$\mathbf{\hat{z}_{t+1}} = \begin{bmatrix} \sqrt{(m_x - \hat{\mu}_{x,t+1})^2 + (m_y - \hat{\mu}_{y,t+1})^2} \\ tan^{-1}\left(\frac{m_y - \hat{\mu}_{y,t+1}}{m_x - \hat{\mu}_{x,t+1}}\right) - \hat{\mu}_{\theta,t+1} \end{bmatrix} \tag{4}$$

Let $\mathbf{Q_{t+1}}$ be the covariance matrix associated with the measurement $\mathbf{z_{t+1}}$.

$$\mathbf{Q_{t+1}} = \begin{bmatrix} \sigma_{r^2} & 0 \\ 0 & \sigma_{\psi^2} \end{bmatrix} \tag{5}$$

We can see from the above equations that the measurement model itself is not a linear one. Hence, we linearize it using Taylor expansion. Let $\mathbf{H_{t+1}}$ be the Jacobian of the measurement model with respect to the state estimate (resulting from the motion model) at time $t+1$.

$$\mathbf{H_{t+1}} = \frac{\partial \mathbf{\hat{z}_{t+1}}}{\partial \hat{\mu}_{\mathbf{t+1}}} = \begin{bmatrix} \frac{\partial \hat{r}_{t+1}}{\partial \hat{\mu}_{x,t+1}} & \frac{\partial \hat{r}_{t+1}}{\partial \hat{\mu}_{y,t+1}} & \frac{\partial \hat{r}_{t+1}}{\partial \hat{\mu}_{\theta,t+1}} \\ \frac{\partial \hat{\psi}_{t+1}}{\partial \hat{\mu}_{x,t+1}} & \frac{\partial \hat{\psi}_{t+1}}{\partial \hat{\mu}_{y,t+1}} & \frac{\partial \hat{\psi}_{t+1}}{\partial \hat{\mu}_{\theta,t+1}} \end{bmatrix} \tag{6}$$

The following equations implement the Extended Kalman Filter, and are stated here only for quick reference. They will be dealt with in detail, in the next few lectures.

$$\mathbf{S_{t+1}} = \mathbf{H_{t+1}} \mathbf{\hat{\Sigma}_{t+1}} \mathbf{H_{t+1}^T} + \mathbf{Q_{t+1}} \tag{7}$$

$$\mathbf{K} = \mathbf{\hat{\Sigma}_{t+1}} \mathbf{H_{t+1}^T} \mathbf{S^{-1}} \tag{8}$$

$$\mu_{\mathbf{t+1}} = \hat{\mu}_{\mathbf{t+1}} + \mathbf{K}(\mathbf{z_{t+1}} - \mathbf{\hat{z}_{t+1}}) \tag{9}$$

$$\mathbf{\Sigma_{t+1}} = \mathbf{\hat{\Sigma}_{t+1}}(\mathbf{I} - \mathbf{K}\mathbf{H_{t+1}}) \tag{10}$$