CSE 483: Mobile Robotics

Lecture by: Prof. K. Madhava Krishna **Lecture #**
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Robot Kinematics (Differential Drive)

1 Introduction

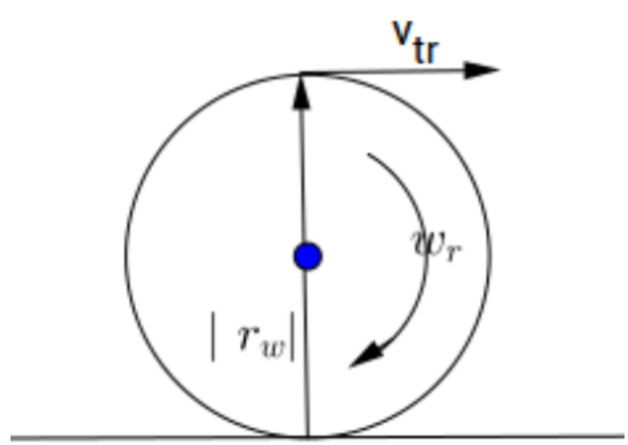
Let X_S be the initial state of the robot (in form of (x, y, θ)) and X_G be the final state of the robot. Now, for a Non-Holonomic system a robot can't get from any arbitrary state X_S to X_G with a single control command.

Such a path may exist using using series of control command executed by the robot.
Let $U_1, U_2, U_3, ..., U_m$ be control command sequence and $X_1, X_2, X_3, ..., X_G$ be the state after executing the respective controls such that $X_t = f(X_{t-1}, U_t)$.

$$\begin{aligned} X_1 =& f(X_S, U_1) \\ X_2 =& f(X_1, U_2) \\ X_3 =& f(X_2, U_3) \\ &. \\ &. \\ &. \\ X_G =& f(X_{m-1}, U_m) \end{aligned}$$

In the following lecture, we will look at how the state of the differential drive robot changes when a particular control U_t is applied.

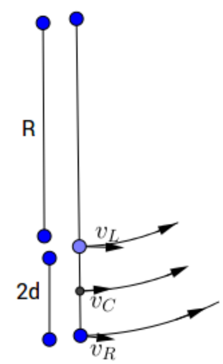
2 Differential Drive



Consider a circular wheel rotating without slipping on a surface. Let b be the point of contact with surface, c be the center, r be the radius and t be the top point. The point of contact with the surface (b) can be considered as Pivot since velocity $\vec{V_{br}} = 0$. Now,

$$\begin{aligned} \vec{V_{tr}} &= \vec{w_r} \times \vec{r_w} \\ \vec{V_{tr}} &= |w_r|[0, 0, -\hat{k}]^T \times 2r_w[0, \hat{j}, 0]^T \\ \vec{V_{tr}} &= |2r_w w_r|[\hat{i}, 0, 0]^T \\ Similarly, \vec{V_{cr}} &= |r_w w_r|[\hat{i}, 0, 0]^T \\ |\vec{V_{cr}}| &= |r_w w_r| \end{aligned}$$

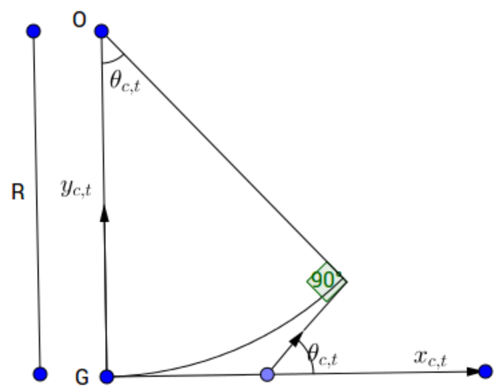
Now, Let's consider a differential drive robot having two such wheels connected by a rigid axial as shown in the figure. R and L are left and right wheel represented as dots. C is the center.



$$|\omega(R + d))| = |v_R|$$
$$|\omega(R - d))| = |v_L|$$

$$\omega = \frac{(v_R - v_L)}{2d}$$
$$v_C = \omega * R$$
$$v_C = \frac{v_R + v_L}{2}$$
$$R = \frac{v_C}{\omega}$$
$$R = \frac{(v_R + v_L)D}{v_R - v_L}$$
$$T = v_C * \Delta t$$
$$\phi = \omega * \Delta t$$

3 Kinematics of differential drive robot in Local Frame



Consider a global frame G, that coincides initially with a frame R fixed on the robot at t=0. the center of the wheel base is

$$X_{c,0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Now, we know that

$$\begin{aligned} v_C &= \frac{v_R + v_L}{2} \\ w &= \frac{v_R - v_L}{2d} \\ X_{c,t}^G &= \begin{bmatrix} x_{c,t} \\ y_{c,t} \\ \theta_{c,t} \end{bmatrix} \end{aligned}$$

$$\begin{aligned} x_c(t) = x_c(0) + \int_0^t v_c \cos(\theta_c) dt \\ = \int_0^t v_c \cos(\omega t) dt \\ y_c(t) = y_c(0) + \int_0^t v_c \sin(\theta_c) dt \\ = \int_0^t v_c \sin(\omega t) dt \\ x_c(t) = \frac{v_c * \sin(\omega t)}{\omega} \\ y_c(t) = -\frac{v_c}{\omega}(\cos(\omega t) - 1) \\ = \frac{v_c}{\omega} - \frac{v_c \cos(\omega t)}{\omega} \end{aligned}$$

$$x_c^2 + (y_c - \frac{v_c}{\omega})^2 = (\frac{v_c}{\omega})^2 = R^2$$
$$x_c^2 + (y_c - R)^2 = R^2$$

This is equation of a circle rotating about 0,R

4 Local frame to Global Frame

Equation that we derived previously are for local frame where robot's initial position is (0,0) and is aligned with the y axis. Now, let X_t^G be state of robot in global origin frame and X_t^R be state of robot in its own frame. Now,

$$X_{t+1}^G = f(X_t^G, v, \omega) = f(X_t^G, v_R, v_L)$$
$$X_{t+1}^R = f(X_t^R, v, \omega) = f(X_t^G, v_R, v_L)$$

But since in robot's own frame starting point is at origin

$$X_{t+1}^R = f(0, v, \omega) = f(0, v_R, v_L)$$

X_{t+1}^R can be found using equations derived in previous section. To translate back the from local frame to global frame, we need to give proper transformation in terms of rotation and translation. Let $T_{R,t}^G$ be that transformation.

$$X_{t+1}^G = T_{R,t}^G * X_t^R$$
$$X_{t+1}^G = \begin{bmatrix} \cos(\theta_c) & -\sin(\theta_c) & x_{c,t} \\ \sin(\theta_c) & \cos(\theta_c) & y_{c,t} \\ 0 & 0 & 1 \end{bmatrix} X_t^{R_t}$$

The End